



Geomorphometry 2025 Perugia

New Tool

for Calculating Land Surface Parameters

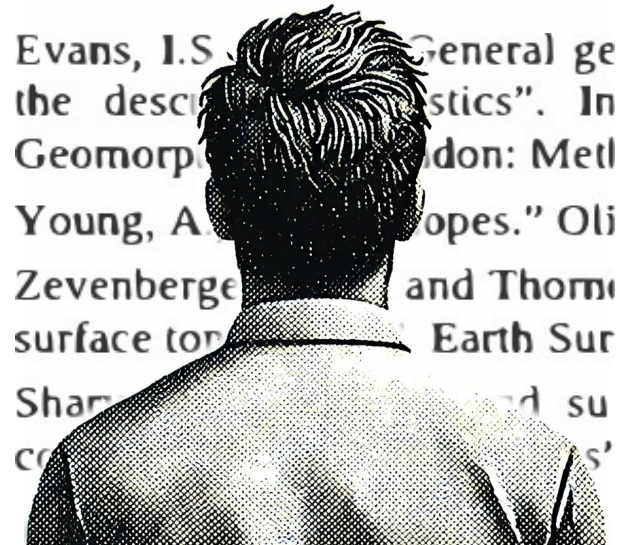


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Yet another Land Surface Parameters calculator?



Motivation

- Third-order LSPs are rarely used
 - Lack of tool support
 - High computational complexity
- Most existing methods lack flexibility
 - Moving window size and shape
 - Grid shape (square or rectangular)
 - No-data handling
- Need for an efficient tool to compute advanced LSPs

Third-order LSPs

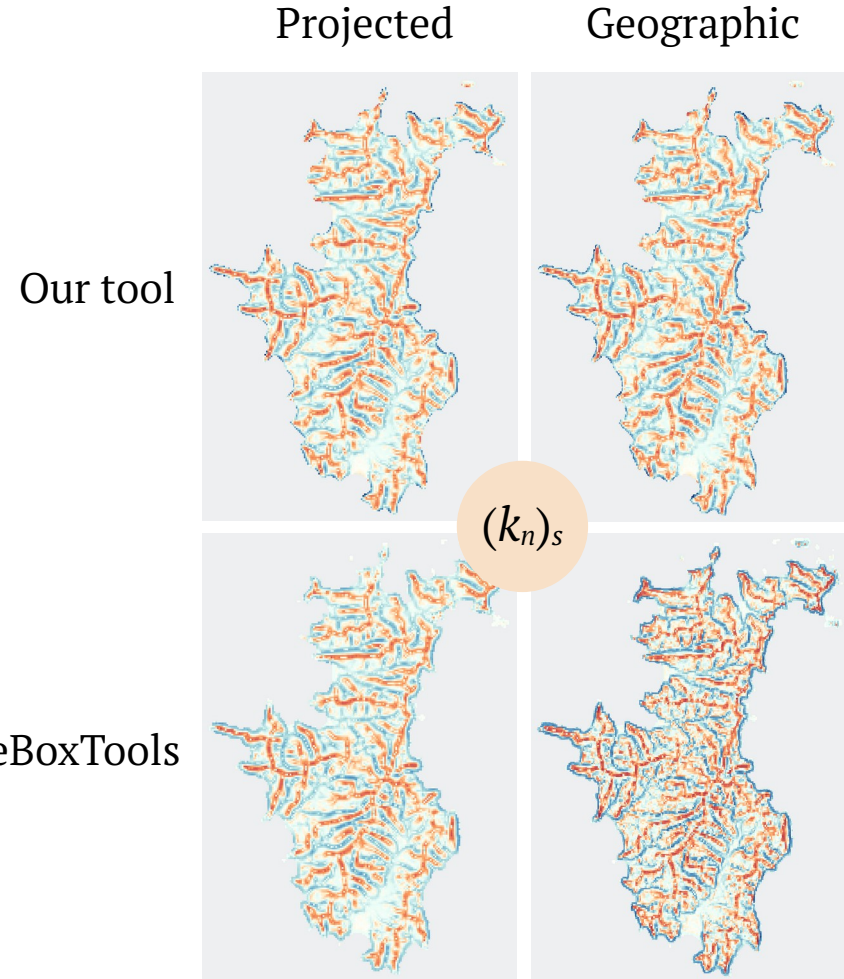
- Changes of curvatures
 - Required for physically based elementary segmentation
 - Physical interpretation
- Partial derivatives up to the third order are needed
 - Minimum required polynomial degree: 3
 - Third-degree bivariate polynomial has 10 coefficients
 - Requires a 5×5 moving window

Computing Partial Derivatives

- Polynomial least-squares fitting
 - Same approach as common methods (2nd degree Evans, 3rd degree Florinsky)
 - Does not use prepared formulas for direct derivative calculation from the grid
 - Uses a general method based on matrix operations instead
 - Uses a 5×5 moving window
- Various orders of polynomials
 - 3rd and 4th degrees supported
 - Higher orders risk of introducing artifacts (Runge's phenomenon)

General Method Advantages

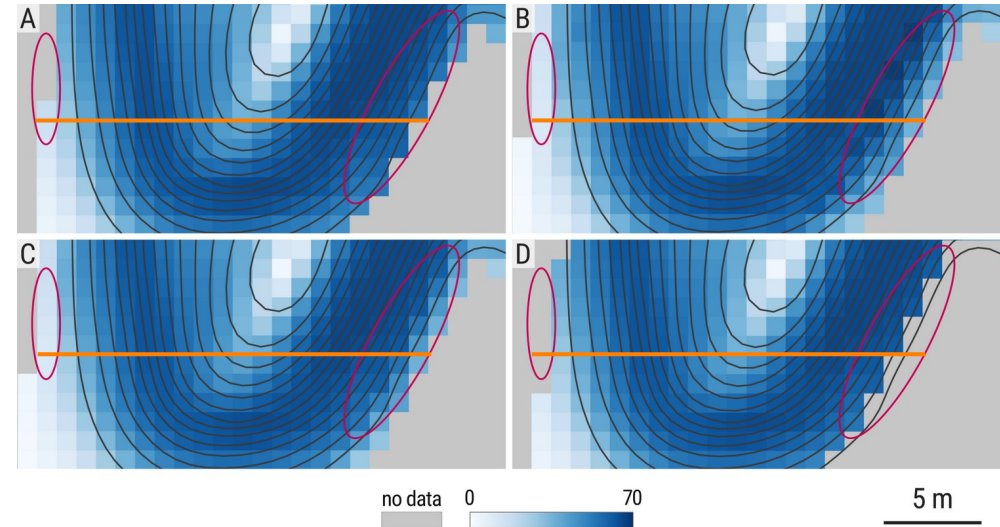
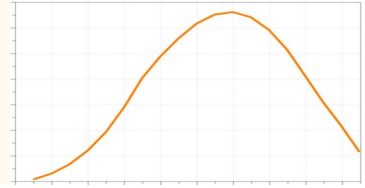
- Easily handles different dx, dy
 - For non-square grid
 - Geographic coordinates
- Same method used for both geographic and projected coordinates



General Method Advantages

- Full neighborhood not required
 - Only enough neighbors needed to fit the polynomial
- Calculations on edge
 - A – calculates correctly
 - B – artifacts, incorrect results
 - C – incorrect results
 - D – does not calculate

Elevation profile



Slope: A – our tool, B – WhiteBoxTools,
C – GRASS GIS (-e), D – GRASS GIS



General Method Disadvantages

■ Computational complexity

- Coordinate powers
- Matrix dot products

■ Optimization is possible

- Precalculation of matrices
- Projected: once per grid, Geographic: once per row
- Missing neighbors reduce its effectiveness

In general, a bivariate polynomial $P(x, y)$ of total degree d , used for fitting to data points (x_i, y_i, z_i) , where z_i represents the elevation at (x_i, y_i) , is expressed as

$$P(x, y) = \sum_{j=0}^n c_j \cdot B_j(x, y) \quad (2)$$

where c represents the unknown coefficients to be determined, and B are the basis functions forming the basis function matrix A

$$A_{ij} = B_j(x_i, y_i) = x_i^{p_i} \cdot y_i^{q_i} \quad (3)$$

The members are the n combinations of exponents p, q such that $p + q \leq d$. A is a matrix of size $(\text{num_points} \times n)$. For example, for a third-degree polynomial and a full 5×5 moving window, A has a size of 25×10 .

The solution for least squares fitting is obtained by solving the normal equations

$$M \cdot c = v \quad (4)$$

where M is the matrix

$$M = A^T \cdot A \quad (5)$$

and v is the right-hand side

$$v = A^T \cdot z \quad (6)$$

The coefficients are computed as

$$c = M^{-1} \cdot v \quad (7)$$

The partial derivatives of a bivariate polynomial $P(x, y)$ are computed by differentiating each term of the polynomial individually. The partial derivative of the j -th basis function with respect to $x^p y^q$ is calculated as

$$\frac{\partial^{p+q} B_j}{\partial x^p \partial y^q} = \binom{p_i}{p} \cdot \binom{q_i}{q} \cdot x^{p_i-p} \cdot y^{q_i-q} \quad (8)$$

or derivative is zero, if $p > p_i$ or $q > q_i$. The contribution to the partial derivative of $P(x, y)$ is obtained by multiplying the derivative of the basis function by its corresponding coefficient c_j

$$\frac{\partial^{p+q} P}{\partial x^p \partial y^q} = \sum_{j=0}^n c_j \cdot \frac{\partial^{p+q} B_j}{\partial x^p \partial y^q} \quad (9)$$

Even though a fourth-order polynomial allows the calculation of fourth partial derivatives, only the first three orders are needed for LSP calculations, so $p + q \leq 3$ in (8) and (9).

Land Surface Parameters

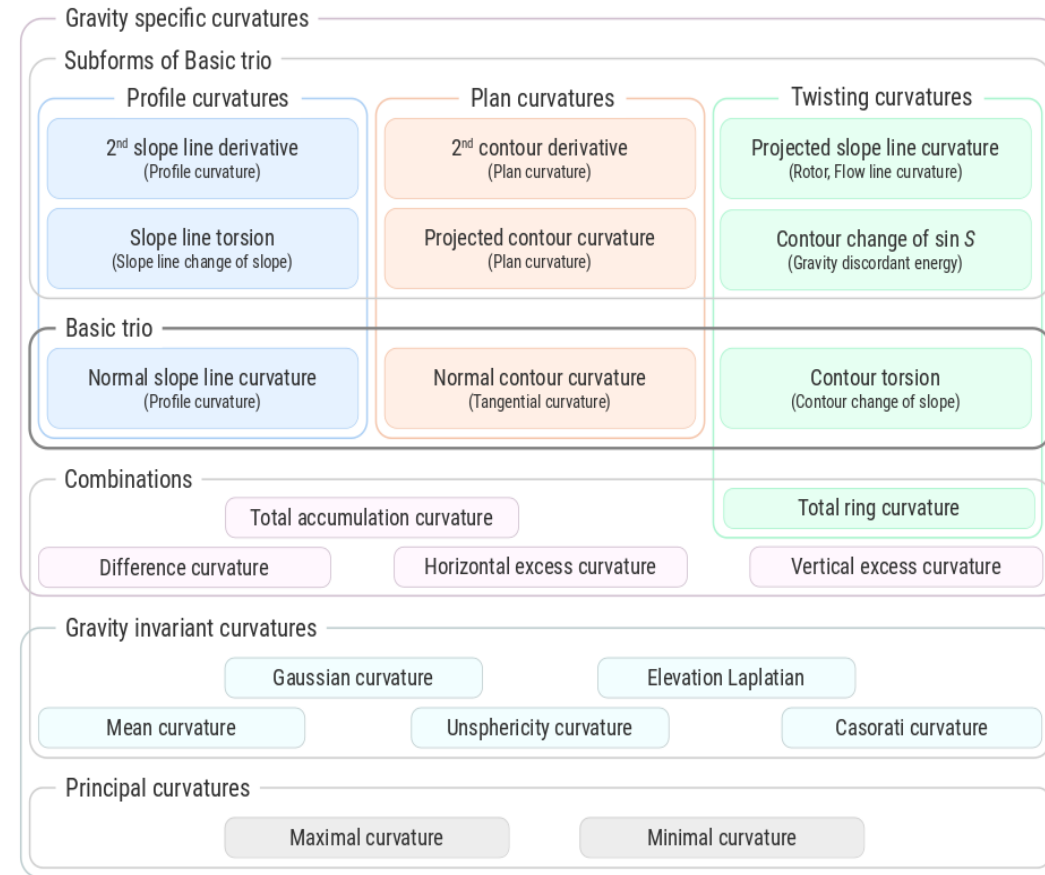
Slope S , Aspect A

■ $\sin S, \sin A, \cos A$

Comprehensive set of curvatures:

Changes of curvatures

- contour change of normal contour curvature $(k_n)_{cc}$
- slope line change of normal contour curvature $(k_n)_{cs}$
- slope line change of normal slope line curvature $(k_n)_{ss}$



Implementation

LSP Calculator

- Command-line tool
- Rust programming language
- Parallelized calculations

```
$ lsp_calculator --input-file dem.tif --output-prefix dem --degree 3 --all
```

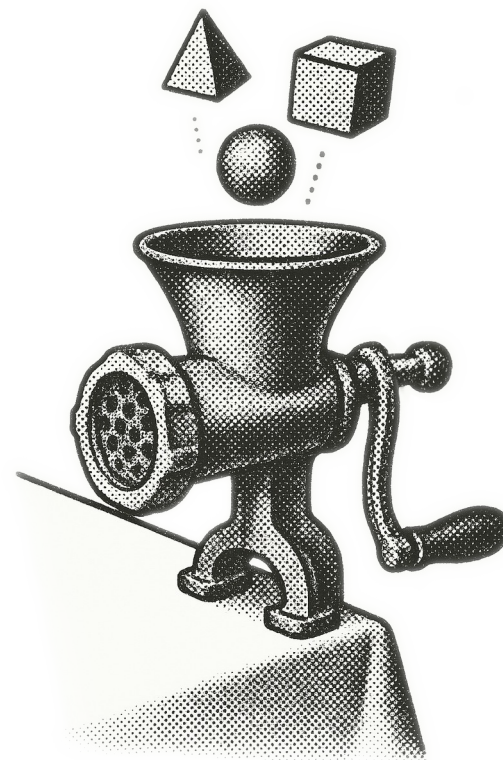
- Github repository

 <https://github.com/xiceph/physical-geomorphometry-tools/tree/main/lsp-calculator>



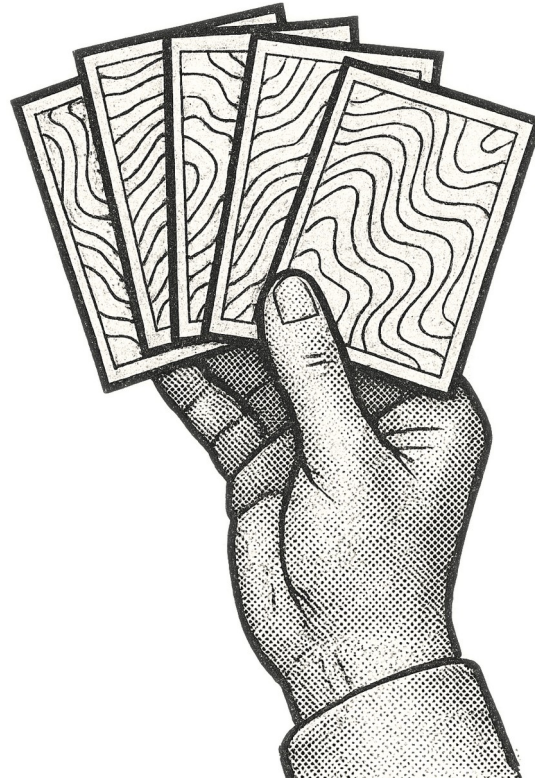
Inputs

- DEM Grid (GeoTIFF)
- Polynomial degree (3 or 4)
- Parameters selection
 - Individual selection
 - Batch selection
- Output name prefix
- Output only partial derivatives
 - Up to 3rd order
 - Fitted elevation output (0th order derivative)

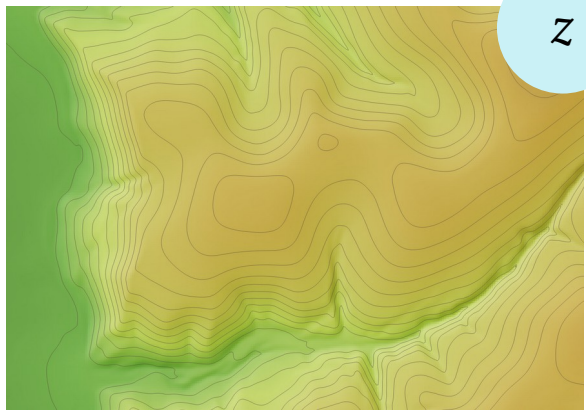


Outputs

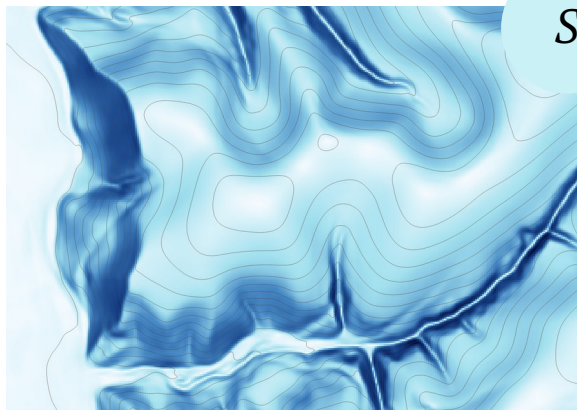
- Multiple grids
 - Number depends on selected parameters
 - All outputs calculated at once
 - GeoTIFF format



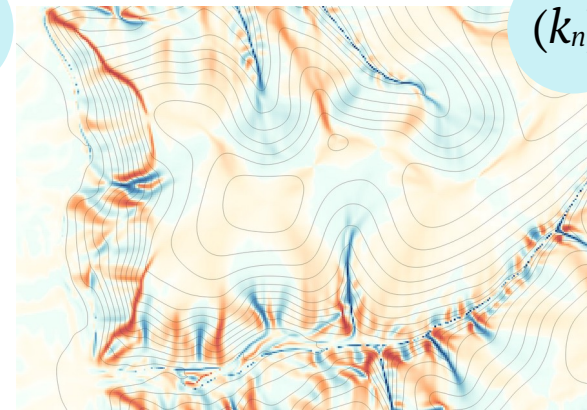
Outputs



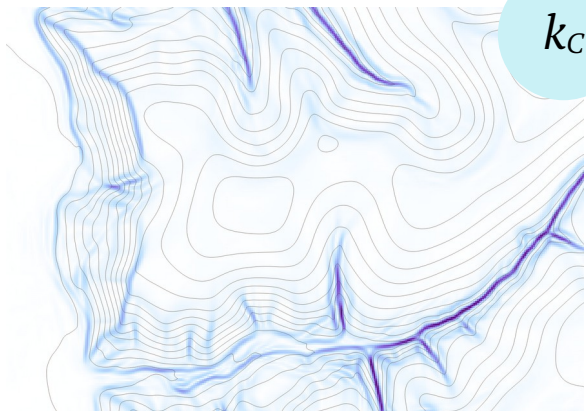
z



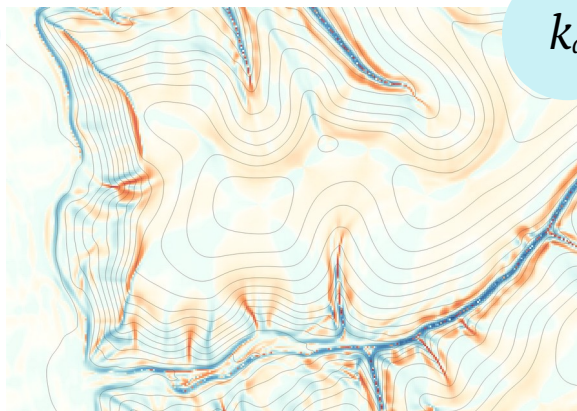
S



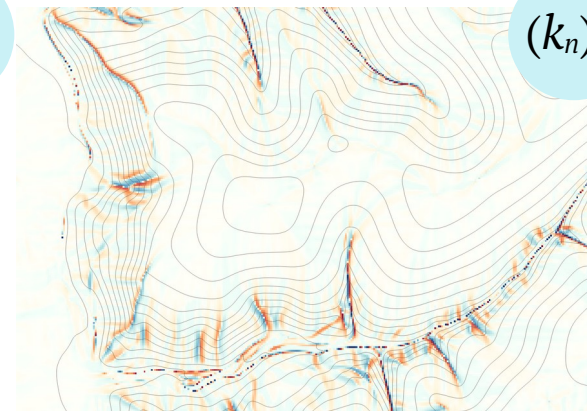
$(k_n)_c$



k_c



k_d




$(k_n)_{cc}$

Web Service


■ We offer web computational service

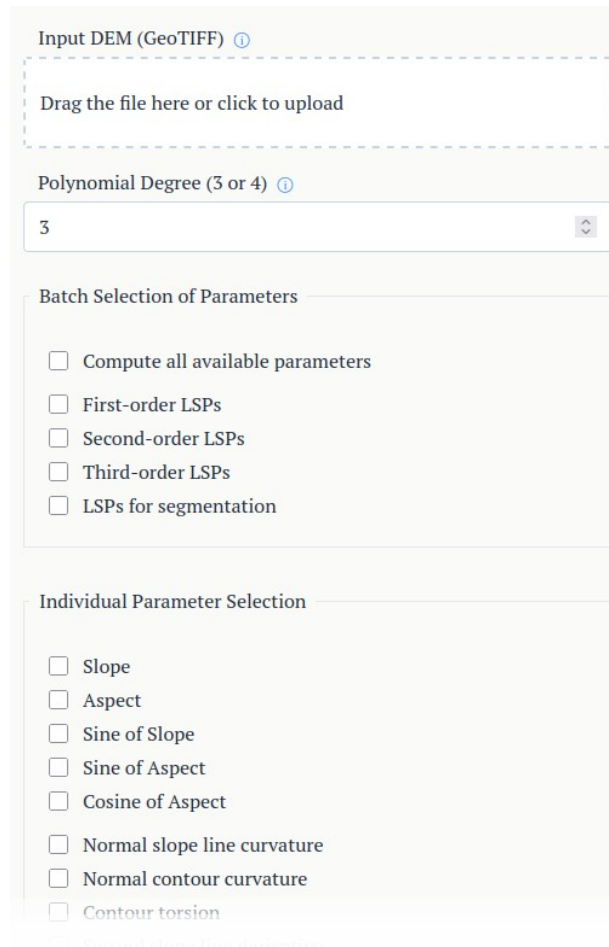
■ No need to install

■ File size and pixels count restriction

■ Currently in test mode 



 https://geomorphometry.fns.uniba.sk/calc-service/lsp_calculator



The screenshot shows a web interface for a geomorphometric calculation service. It features a file upload section for a DEM (GeoTIFF) file, a dropdown menu for selecting the polynomial degree (set to 3), and two sections for parameter selection. The 'Batch Selection of Parameters' section includes checkboxes for 'Compute all available parameters', 'First-order LSPs', 'Second-order LSPs', 'Third-order LSPs', and 'LSPs for segmentation'. The 'Individual Parameter Selection' section includes checkboxes for 'Slope', 'Aspect', 'Sine of Slope', 'Sine of Aspect', 'Cosine of Aspect', 'Normal slope line curvature', 'Normal contour curvature', and 'Contour torsion'. A footer note mentions the service is in test mode.

Input DEM (GeoTIFF) ⓘ

Drag the file here or click to upload

Polynomial Degree (3 or 4) ⓘ

3

Batch Selection of Parameters

- ☐ Compute all available parameters
- ☐ First-order LSPs
- ☐ Second-order LSPs
- ☐ Third-order LSPs
- ☐ LSPs for segmentation

Individual Parameter Selection

- ☐ Slope
- ☐ Aspect
- ☐ Sine of Slope
- ☐ Sine of Aspect
- ☐ Cosine of Aspect
- ☐ Normal slope line curvature
- ☐ Normal contour curvature
- ☐ Contour torsion

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